

# **A Sensitivity-Based Linear Approximation Method to Estimate Time-Dependent Origin-Destination Demand in Congested Networks**

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**1 ABSTRACT**

2           This paper presents a bi-level optimization problem to estimate offline time-dependent  
3 origin-destination (TDOD) demand based on link flows and historical OD matrices. The upper-  
4 level problem aims to minimize the summation of errors in both traffic counts and OD demand.  
5 Conventionally, OD flows are linearly mapped to link flows using the assignment matrix  
6 proportions obtained from the dynamic traffic assignment (DTA), which is typically formulated  
7 as the lower-level problem. However, the linear relationship may be invalid when congestion  
8 builds up in the network, resulting in a nonlinear relation between OD flows and link flows. The  
9 nonlinearity may lead to a converged solution that is far away from the global optimum.  
10 Therefore, an accurate solution should be able to relax the linear assumption and to consider the  
11 effect of other OD flows on the links' traffic volumes. In this study, a solution method based on  
12 the sensitivity of assignment proportions to OD flows has been proposed and applied requiring a  
13 number of further assignments. To overcome the extra computational requirement, we propose a  
14 heuristic method to exclude the extra number of assignments and to improve the performance of  
15 the solution. The proposed algorithm is applied to multiple networks of varying size and  
16 congestion level. Results demonstrate the efficiency of the proposed method for congested  
17 networks with less computation time.

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19  
20

21 **Keywords:** Time-dependent Origin-Destination Demand Estimation, Dynamic Traffic  
22 Assignment, Bi-level Optimization, Congested Network

## 1 INTRODUCTION

2 Time-dependent origin-destination (TDOD) demand is a necessary input to simulated-  
3 based dynamic traffic assignment (DTA) models. The TDOD demand often obtained from static  
4 demand models usually contains errors, which are transferred into the network outputs resulting  
5 in unreliable planning and operational outcomes. Therefore, the TDOD demand estimation  
6 (adjustment) problem is recognized as a critical part of DTA models calibration. Significant  
7 progress has been made towards formulating and solving the TDOD demand estimation problem  
8 in the past few decades (1–6). Various stochastic (7, 8) and deterministic (9–13) solution  
9 algorithms for solving the TDOD estimation problem have been proposed in the literature. Since  
10 the number of unknown variables is larger than the number of equations in the TDOD estimation  
11 problem, the accuracy of the initial OD demand matrix significantly affects the convergence of  
12 the solution algorithm. A few studies explored approaches to provide more reliable initial OD  
13 demand matrices taking into considerations the day of the week and weather conditions (14–16).  
14 Moreover, with the advent of new technologies for traffic data collection, a practical solution to  
15 improve the results of the TDOD demand estimation is using various types of information rather  
16 than link flows only (17, 18).

17  
18 Estimation of TDOD demand in congested networks is still challenging particularly for  
19 large-scale networks. This is due to the fact that congestion may exert a non-linear effect on the  
20 relationship between OD flows and link flows (19, 20). When spillback forms, link flow  
21 measurements are no longer solely dependent on demand flows; rather, supply-side  
22 characteristics become important due to link capacity constraints. This problem results in further  
23 methodological development that focuses on using stochastic solutions such as simultaneous  
24 perturbation stochastic approximation (SPSA) (21). Since methods of this type are assignment-  
25 matrix-free, they need to compute an approximated gradient in the solution which leads to a  
26 number of additional iterations. The increased number of required iterations reduces the  
27 applicability of the methods for large-scale problems where the DTA is computationally  
28 intensive (22, 23).

29  
30 Recent studies by Toledo and Kolechkina (11) and Frederix et al. (24–26) stressed the  
31 need of relaxing the fixed proportional assignment assumption in mapping demand flows to link  
32 flows in congested networks. Toledo and Kolechkina (11) proposed a method based on the linear  
33 approximation of the assignment matrix. They relaxed the conventional assumption of fixed  
34 assignment proportions by considering the marginal effect of the OD flow passing through the  
35 links. Making the assignment proportions only dependent on the relevant OD flows helps to  
36 reduce the number of estimated parameters. However, this assumption may not hold in congested  
37 networks since the variation of OD flows can have significant impacts on the entire assignment  
38 proportions particularly through intersections and spillbacks. Frederix et al. (26) proposed an  
39 approach that the sensitivity of assignment proportions to OD flows are considered in the TDOD  
40 demand estimation problem, taking into account that link flows are not separable. The sensitivity  
41 analysis of assignment proportions to the OD flows requires performing extra assignments. They  
42 used the Marginal Computation (MaC) (27) technique to perform extra assignments in more  
43 efficient way. The MaC algorithm keeps the base simulation result and re-calculates only the part  
44 of the network that changes in the sensitivity analysis. By applying this method, they could  
45 substantially decrease computation time of dynamic network loading (DNL) to perform the

1 sensitivity analysis of assignment proportions. In another study (25), Frederix et al. suggested a  
 2 space decomposition technique. They presented a new hierarchal approach that focuses on  
 3 separating the network into congested and uncongested sub-networks. The MaC algorithm was  
 4 then applied to the congested sub-network only. Since computing sensitivity for all OD pairs is  
 5 computationally intensive, any proposed algorithm to be applied on real-size networks should  
 6 avoid performing extra computations (28, 29). In fact, previous studies by Frederix et al.(24–26)  
 7 highlighted the problem of computing sensitivity for all OD pairs in medium or large-scale  
 8 networks. The authors proposed MaC algorithm to execute extra assignments more efficient.  
 9 Although the idea is interesting, the proposed solution cannot be readily obtained for three  
 10 reasons. First, the number of OD pairs grows rapidly in a larger network. As a result, computing  
 11 the sensitivity of all OD pairs remains intractable even when an efficient DNL model is used.  
 12 Second, errors from the simplification of the DNL model may considerably distract the  
 13 convergence of the solution. Finally, the application of the MaC algorithm requires to improve  
 14 the DNL models, which may be difficult to implement for the majority of the exiting DNL/DTA  
 15 models.

16  
 17 This paper relaxes the conventional assumption on the fixed assignment proportion in  
 18 mapping OD flows to link flows in the TDOD demand estimation problem. A heuristic algorithm  
 19 is proposed to compute the sensitivity of assignment proportions to OD flows more efficiently.  
 20 The method considers the correlation between OD flows and link flows and applies a systematic  
 21 approach to determine the OD flows that have higher tendency to vary during the TDOD demand  
 22 estimation procedure. The paper demonstrates that the proposed method is applicable in real-size  
 23 networks.

24  
 25 The remaining of the paper is organized as follows. The next section describes the  
 26 conventional formulation problem and the required enhancements in the TDOD demand  
 27 estimation for large-scale congested networks. The third section discusses the overall steps to  
 28 obtain the optimal solution. Section four summarizes the modeling results on studied networks.  
 29 The last section draws some conclusions and suggests future research directions.

## 30 **PROBLEM FORMULATION**

31 The TDOD demand estimation problem can be formulated as a bi-level optimization  
 32 problem (3) . The upper-level is the travel demand matrix estimation problem and the lower-  
 33 level is a DTA model producing detailed dynamic traffic patterns. Traditionally, two objectives  
 34 are considered in the upper-level. The first objective is to minimize the deviation between  
 35 observed link flows and simulated link flows, while the second objective is to minimize the  
 36 deviation between the priori demand and the estimated demand. Mathematically, this problem  
 37 can be formulated as follows:

$$38 \begin{aligned} & \text{minimize } Z(X) = [w \cdot f_1(X, \hat{X}) + (1 - w) \cdot f_2(Y, \hat{Y})] \\ & Y = \text{DTA}(X) \end{aligned} \quad (1)$$

39  
 40 where  $f_1$  and  $f_2$  are deviation functions,  $f_1$  measures the error between the estimated OD demand  
 41  $X$  and the initial OD demand  $\hat{X}$ ,  $f_2$  measures the error between the estimated  $Y$  and observed  
 42 link traffic counts  $\hat{Y}$ .  $w$  represents the weight on the deviation from the initial matrix, and

1 DTA( $X$ ) represents link volumes obtained from the dynamic user equilibrium (DUE) model. The  
 2 equation that connects the lower level problem to the upper level is conventionally formulated as  
 3 follows:

$$4 \quad y_a^h = \sum_{t=1}^h \sum_{i \in I} p_{a,i}^{h,t}(X) x_i^t \quad (2)$$

5

6 where

7  $y_a^h$ : link flow at link  $a$  during time period  $h$ ,

8  $x_i^t$ : flow between OD pair  $i$  departing in time period  $t$ ,

9  $p_{a,i}^{h,t}$ : proportion of  $x_i^t$  that passes link  $a$  during time period  $h$ ,

10  $X$ : vector of all OD flows  $x_i^t$ ,

11  $I$ : OD pairs set.

12

13 Frederix et al. (24, 26) suggested using the Taylor approximation to specify the linear  
 14 relationship of link flows at the current demand flow ( $X^*$ ). By substituting Eq.2 with the first-  
 15 order Taylor approximation the following equation is obtained:

16

$$\begin{aligned} y_a^h &= \sum_{t=1}^h \sum_{i \in I} \left[ p_{a,i}^{h,t}(X^*) x_i^t + \frac{dy_a^h(X^*)}{dx_i^t} (x_i^t - x_i^{t*}) \right] \\ &= \sum_{t=1}^h \sum_{i \in I} \left[ p_{a,i}^{h,t}(X^*) x_i^t + \frac{d(\sum_{\tau=1}^h \sum_{i \in I} p_{a,i}^{h,\tau}(X^*) x_i^{\tau*})}{dx_i^t} (x_i^t - x_i^{t*}) \right] \\ &= \sum_{t=1}^h \sum_{i \in I} \left[ p_{a,i}^{h,t}(X^*) x_i^t + \left( \sum_{\tau=1}^h \sum_{i \in I} \frac{d(p_{a,i}^{h,\tau}(X^*))}{dx_i^t} x_i^{\tau*} \right) (x_i^t - x_i^{t*}) \right] \end{aligned} \quad (3)$$

17 The second term of the Taylor approximation (Eq.3) incorporates the sensitivity of the  
 18 assignment matrix to changes in the OD flows. As mentioned, computing the exact Jacobian  
 19 vector in the second term is impossible in median or large-scale networks. In the next section,  
 20 we propose an algorithm that estimates the second term of Eq.3 more efficiently, making it  
 21 applicable for large-scale network applications.

## 22 METHODOLOGY

23 Here we propose an iterative sensitivity-based linear approximation algorithm as follows:

24 *Step 0.* Superimpose an initial static OD matrix by a general time profile;

25 Set  $I' = \{\}$ ,  $n=0$ .

26 *Step 1.* Load demand to the network to obtain assignment matrix,  $p_{a,i}^{h,t}$ .

27 *Step 2.* Calculate the sensitivity of all observed links to the OD pairs in set  $I'$ .

28 *Step 3.* Estimate the TDOD demand matrices with link volumes given by Eq.3.

29 *Step 4.* Determine OD pairs that changed considerably in previous iterations and  
 30 insert them in  $I'$ .

1           Step 5. Update current demand,  $X^*$ , with estimated demand  $X^n$ .

2           Step 6. If the termination criteria is met, stop. Otherwise, go to step 1 and  $n=n+1$ .

3  
4           An initial estimate of the OD matrix for each time interval is set in step 0. Also  $I' \in I$  is  
5 defined as a set containing all the OD pairs that change considerably in the previous iterations. In  
6 step 1, the current demand is loaded to the network resulting in the assignment matrix,  $p_{a,i}^{h,t}$ .  
7 Then, the second term of Eq.3 is calculated in step 2. Since computing sensitivity of all observed  
8 links for all OD flows is computationally intensive, sensitivity analysis in step 2 is only applied  
9 to the OD pairs that were included in  $I'$ . In step 3 the upper-level problem is solved using a  
10 gradient decent method. In step 4, by comparing the new estimated demand  $X^n$  with the previous  
11 value  $X^{n-1}$ , the OD pairs that have more tendency to vary are determined and inserted to the set  
12  $I'$ . Then, in step 5, the new demand is updated. Note that in the first iteration ( $n=0$ ) where the  
13 sensitivity computation cannot be performed, the new demand is not updated. The results  
14 obtained are checked in step 6 against the termination criteria and the procedure would continue  
15 if it is not met.

16           A heuristic solution algorithm is employed to reduce the computation time for the sensitivity  
17 analysis. The following are several reasons that the variations of many OD pairs may not have a  
18 significant effect on the TDOD demand estimation:

- 19           - In large-scale TDOD demand matrices, many OD pairs have small values. The impact of  
20 these individual OD pairs are insignificant on the network and hence, the sensitivity  
21 analysis for each of them is unnecessary.
- 22           - Flows associated with many OD pair do not cross the congested area. As a result, the new  
23 demand can be accurately estimated using the assignment proportions as in a traditional  
24 method.
- 25           - Within a few iterations a number of OD demand flows converge and would not change  
26 considerably in the subsequent iterations. Thus, these OD pairs are excluded from the  
27 sensitivity analysis.

28           Therefore, a set of candidate OD pairs is defined on which the sensitivity analysis is applied.  
29 According to the proposed method, in iteration 0, there is no OD pair in the set  $I'$ , indicating that  
30 in this iteration, Eq. 3 is simplified to Eq. 2. However, in the subsequent iterations, the qualified  
31 OD pairs based on the above identification procedure are selected and moved to the set  $I'$ .

### 32 33 SOLUTION ALGORITHM

34  
35           Here we apply the commonly used ordinary least square (OLS) approach as the  
36 functional form of the component functions  $f_1$  and  $f_2$  in Eq.1. The link traffic volumes are  
37 calculated as expressed in Eq.3 based on the assignment proportions obtained from the DTA  
38 model. Different types of exact and heuristic methods are available and can be employed to solve  
39 the optimization problem in Eq.1. Here we solve the problem iteratively using the gradient  
40 descent method. The method requires calculation of the gradient direction and step size to  
41 update the OD demand matrix in each iteration using the following equation:

$$42 \quad X^{(K)} = X^{(K-1)} + \frac{\partial Z}{\partial X} \lambda \quad (4)$$

1 The gradient is determined as follows:

2

$$\begin{aligned}
 \frac{\partial Z}{\partial x_i^t} &= \frac{\partial}{\partial x_i^t} \left( (1-w) \sum_h \sum_a (y_a^h - \hat{y}_a^h)^2 + w \sum_i \left( \sum_t x_i^t - \hat{x}_i^t \right)^2 \right) \\
 &= 2(1-\omega) \left( \sum_{t=1}^h \sum_{i \in I} p_{a,i}^{h,t}(X) x_i^t + \sum_{\tau=1}^h \sum_{i \in I} \frac{d(p_{a,i}^{h,\tau}(X^*))}{dx_i^t} x_i^{\tau*} (x_i^t - x_i^{\tau*}) - \hat{y}_a^h \right) \\
 &\left( p_{a,i}^{h,t}(X) + \sum_{\tau=1}^h \sum_{i \in I} \frac{d(p_{a,i}^{h,\tau}(X^*))}{dx_i^t} x_i^{\tau*} \right) + 2\omega(x_i^t - \hat{x}_i^t)
 \end{aligned} \tag{5}$$

3

4 We find the step size ( $\lambda$ ) analytically by determining the moving direction in each iteration:

5

$$\Gamma^k = \min_{\lambda} Z(x_i^t + \frac{\partial Z}{\partial x_i^t} \lambda) \tag{6}$$

6

7 To solve the sub-problem as shown in Eq.6, the golden section algorithm, as a line search  
8 minimization solution, is applied.

## 9 NUMERICAL RESULTS

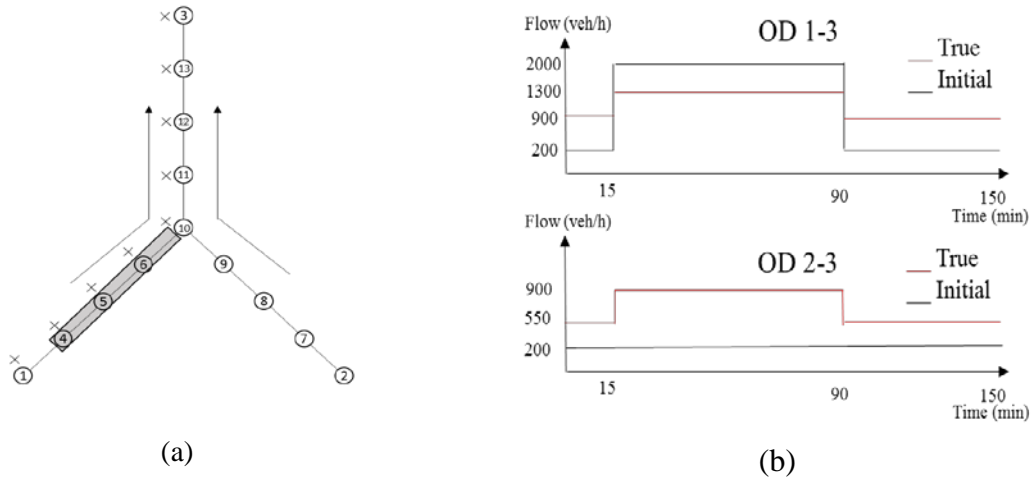
10 In this section, different linear models relating OD flows to link flows are compared. In  
11 the conventional model (Eq.2) the assignment proportions map the two flows on each other  
12 while in the new approach, sensitivity of demand flows to link flows are also taken into account.  
13 The sensitivity analysis can be performed either by using an exact solution method or the  
14 proposed heuristic solution algorithm. The comparison is first conducted in the same network  
15 previously used in the literature (24, 26) for the sake of consistency and reproducibility. We,  
16 then, apply the method on a real network introduced as a benchmark in a recent study (2) to  
17 demonstrate its performance in more complex configurations. The algorithm is developed in  
18 Python and applied using AIMSUN simulation-based model (30).

19

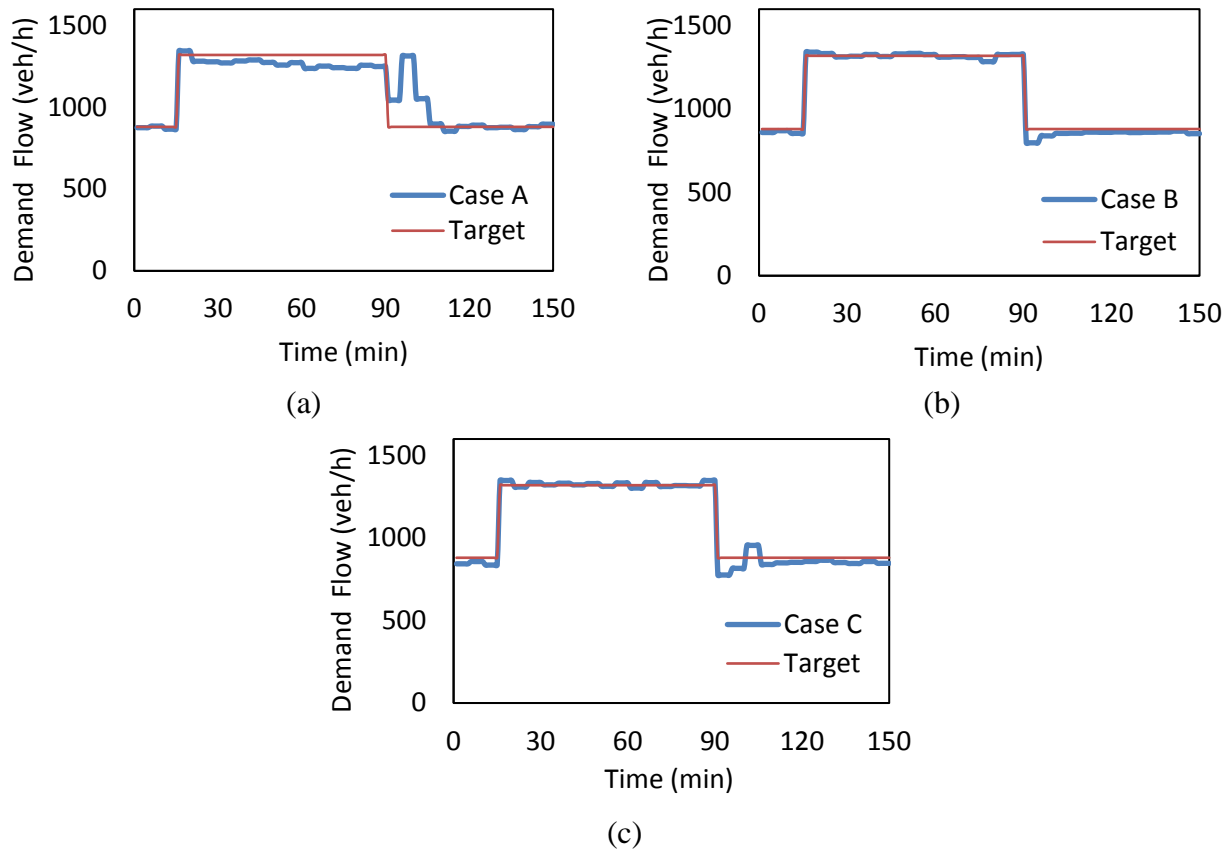
### 20 *Methodology verification*

21 Figure 1 shows the study network consisting of three branches that merge in node 10.  
22 Each branch is divided into four equal one-way links with the same physical characteristics.  
23 Each link has a length of 1 km, capacity of 1,800 vehicles per hour, free flow speed of 90 km/h  
24 and jam density of 140 vehicles per kilometer. Node 1 and node 2 are origins and node 3 is the  
25 destination. The target (true) demand is depicted in Figure 1(b). After 15 minutes, due to a jump  
26 in the OD flow, a queue propagates in the left branch. The length of the queue increases till  
27 minute 95. The maximum queue length is demonstrated by a rectangle in Figure 1(a). Due to the  
28 smaller OD flow in the right branch, the queue length is quite short. In the third stage of the  
29 network, i.e. after minute 95, the OD flow decreases to the initial stage and therefore, the traffic  
30 queue gradually dissipates by the end of the simulation. The traffic volumes obtained by loading  
31 the true demand to the network are considered as target volumes, which are captured by the  
32 detectors shown in Figure 1(a) with an 'X'.

33

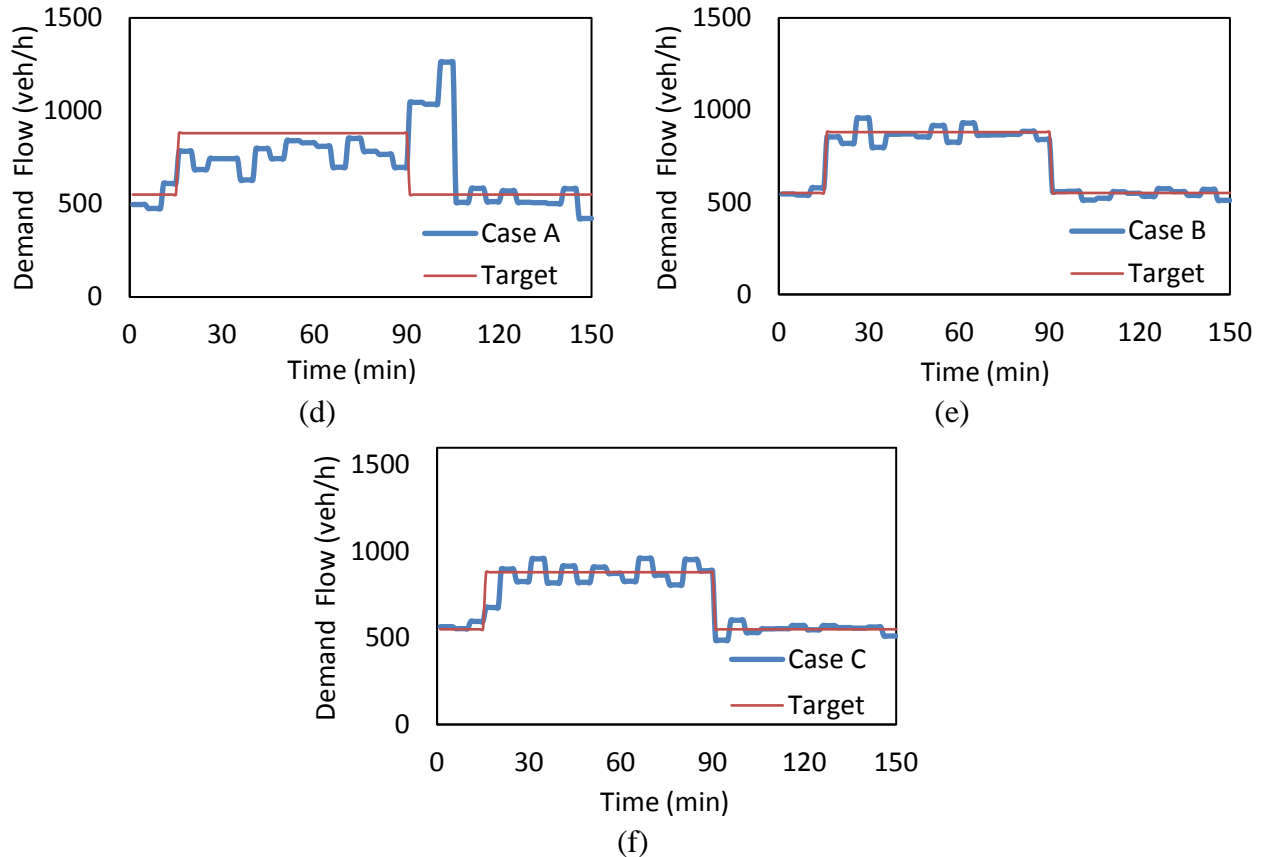


1 **FIGURE 1.** (a) Schematic illustration of study network. (b) Initial and true demand profiles (26)  
 2  
 3



4 **FIGURE 2** Comparing the estimated demand using different solution approaches. (a), (b) and  
 5 (c) show the demand profiles of origin 1 to destination 3.  
 6





**FIGURE 3** Comparing the estimated demand using different solution approaches. (d), (e) and (f) show the demand profiles of origin 2 to destination 3.

The assumed initial O-D matrix profile is shown in Figure 1(b). Three different approaches are applied to solve the TDOD estimation problem for this network:

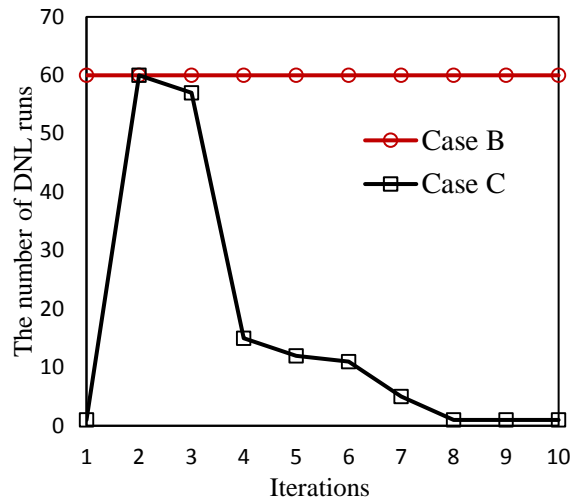
Case A) Using the conventional linear model (Eq.2)

Case B) Using the first order Taylor approximation (Eq.3) - computing the exact value of the Jacobian vector.

Case C) Using the first order Taylor approximation (Eq.3) - computing a part of the Jacobian vector.

Figure 2 and 3 show the estimated demand using different solution approaches. The target demand is also presented as a point of reference for both OD pairs. The conventional linear process (case A) underestimates the congested state of the network and overestimates demand especially for the demand flow from node 2 to node 3. The approach requires low computation time and during the estimation, the DTA model is run for a few times only. However, this approach is more suitable for uncongested networks as discussed previously. The model errors at the beginning and the end of the simulation remain small, which suggests that the model could accurately reproduce the target demand. However, by increasing the congestion in the network, the accuracy cannot be maintained. In case B, the exact sensitivity term is computed for all OD pairs during the entire simulation. The computation is intensive, however, the estimated model can capture the true demand with some noise. In case C, the results are quite

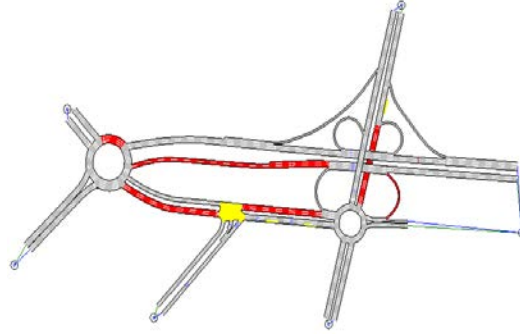
1 close to those in case B. The small difference is due to the simplification of the sensitivity  
 2 computation in the third approach. Figure 4 shows the required number of DNL model runs for  
 3 10 iterations. In case B, the number of extra assignments remains constant in each iteration  
 4 because the full sensitivity term is required to be calculated. However, in case C, the number of  
 5 extra assignments varies based on the number of OD pairs in set  $I'$ . In the first iteration, there is  
 6 no nominated OD in set  $I'$  and hence, the resulting estimated OD demands are compared with  
 7 their original values. Since the values of all OD pairs are changed over 5%, they become  
 8 nominated in iteration 2 and enter set  $I'$ . The process continues in the next iteration where the  
 9 estimated demand values are compared with their previous ones to renew set  $I'$ . In iteration 4,  
 10 most of the OD pairs converge and thus, are excluded from set  $I'$ . The remaining OD pairs  
 11 mainly belong to the time intervals where sudden changes in demand occur (around minute 15  
 12 and 90). After a few iterations, these OD pairs also converge. For the study network, the exact  
 13 solution (case B) requires 600 DNL model runs while in case C this number is reduced to 164.  
 14



15  
 16 **FIGURE 4.** The number of DNL runs in case (B) and (C).

17  
 18 *Methodology implementation*

19  
 20 The methodology is further applied to a larger network to gain more insights into the  
 21 general performance of the proposed algorithm. The study network includes complex traffic  
 22 elements such as two roundabouts and adaptive signal controls. The simulation was conducted  
 23 for one hour divided into 10-min time intervals leading to six time intervals. In this numerical  
 24 study, 6 links are considered to have detectors. The corresponding link flows have been derived  
 25 from assignment of target (true) TDOD matrix treated as target counts. AIMSUN dynamic user  
 26 equilibrium (DUE) was used. Due to the congestion or to the distance between OD pairs, the  
 27 trips for vehicles that departed at a given time interval in the network are not necessarily  
 28 completed within the same time interval.  
 29



**FIGURE 5.** Schematic illustration of the study network. Red links represent target links.

Figure 6 illustrates the initial, target and estimated demand profiles in one hour simulation. The target demand increases in the second and third time intervals and then reduces again. This variation causes queue propagation in on-ramps and thus, the queue spills back to the adjacent links. In the last few time intervals, the congestion gradually dissipates. To explore the performance of the TDOD demand estimation process, a fixed demand pattern, as shown in Figure 6 is considered as an initial demand. The quality of the initial demand affects the TDOD demand estimation performance. The initial demand flows are obtained randomly perturbing the target matrix in the range of  $\pm 30\%$ . A number of statistical measures can be employed to evaluate the performance of TDOD estimation approaches, measuring the estimated outputs. Mean absolute error (MAE) and root mean square error (RMSE) are used to quantify the deviation between measured and estimated link flows:

$$MAE = \frac{1}{N} \sum_{n=1}^N |Y_n - \hat{Y}_n| \quad (7)$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^N (Y_n - \hat{Y}_n)^2} \quad (8)$$

where

$N$ : is the number of observations,

$\hat{Y}_n$ : is an observation, and

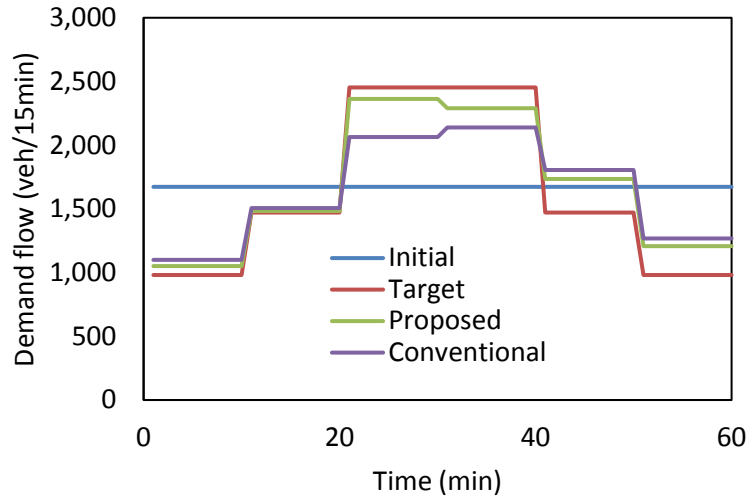
$Y_n$ : is a simulated value at time  $n$ .

Table 1 shows the goodness of fit before and after the TDOD demand estimation process. In general, a significant reduction in RMSE and MAE values are obtained after applying TDOD demand estimation. The performance of the two methods are also compared. Although the conventional method reduced the gap between the simulated link flows and the target values, the quality of the solution in terms of the estimated link flows and demand flows obtained from the proposed algorithm is higher. For the TDOD demand estimation problem, the following termination criterion is used where  $k$  is the number of iterations:

$$\text{Relative gap} = \frac{RMSE_K - RMSE_{k-1}}{RMSE_K} < 0.05 \quad (9)$$

**TABLE 1.** RMSE of link traffic volumes for the TDOD estimation results.

	RMSE	MAE
Initial	117.9	87.8
Conventional approach (Case A)	37.3	24.6
Proposed approach (Case C)	10.5	8.3

**FIGURE 6.** Initial, target and estimated demand profiles.

To further investigate the efficiency of the proposed approach, we implement the OD estimation method on a benchmark network (2). The observed traffic data are obtained from 10 detectors in various locations across the network. The simulation study is conducted for the morning peak (8:00 AM to 9:00 AM) in 15-min time intervals. The number of OD pairs are 90 in each time interval which indicates that the total number of estimated OD pairs is equal to 360 (31).

The TDOD estimation problem is applied to the network with three demand scenarios: uncongested, moderately congested, and heavily congested. The DTA model is run using the AIMSUN simulator outputting the assignment proportion matrices. Based on the obtained matrices, the bi-level problem is solved using the gradient descent method.

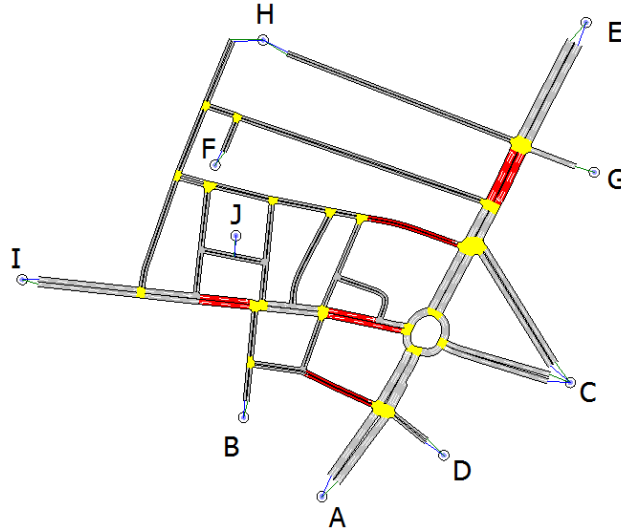
To derive the target demand flows, a uniform random component in the range of  $\pm 40\%$  is added to the initial demand and multiplied by three factors to produce different congestion levels, mathematically expressed as follows:

$$X = 0.8\hat{X} \cdot [0.6 + 0.8 \alpha] \quad \alpha \sim U(0,1) \quad (10)$$

$$X = 1.2\hat{X} \cdot [0.6 + 0.8 \alpha] \quad \alpha \sim U(0,1) \quad (11)$$

$$X = 1.5\hat{X} \cdot [0.6 + 0.8 \alpha] \quad \alpha \sim U(0,1) \quad (12)$$

Table 2 illustrates several performance measures of the network with different OD demands. Note that the network's critical density can be much lower than the link critical density specifically for a heterogeneous network (32–34).



**FIGURE 7.** Schematic illustration of the study network. Red links represent target links.

**TABLE 2.** Compare performance measures of the network in different congestion levels.

	Uncongested	Moderately congested	Heavily congested
Flow (veh/h)	2,535.1	3,956.9	4,908
Speed (km/h)	24	20.21	15.46
Delay time (sec/km)	98.91	136.88	247.65
Density (veh/km)	5.75	11.38	21.87

Table 3 shows the RMSE and MAE before and after the TDOD demand estimation process for both the conventional linear method and the proposed method. It can be seen that the performance measures are favorable when applying the conventional method to the uncongested network. By increasing the congestion level, the proposed method provides a more reliable estimation of OD demand demonstrating the applicability of the proposed method on congested networks.

The termination criterion in this case study is expressed in Eq.9. Ten replications for each assignment were employed to address the stochasticity of the simulation. The DTA assignment interval was 5 minutes and all the graphical elements were closed. The models were run on a 64-bit machine using 64 GB memory and an Intel Xeon 12 CPUs @ 3.5 GHz processor. The computation times for the entire OD estimation procedure were about 20, 35 and 40 minutes for the uncongested, moderately congested, and heavily congested respectively.

**TABLE 3.** RMSE of link traffic volumes for the TDOD estimation results.

	Uncongested		Moderately congested		Heavily congested	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
Initial	32.28	26.50	52.00	30.5	75.42	47.82
Conventional approach	17.99	13.25	22.65	14.37	36.59	25.57
Proposed approach	13.08	10.10	19.11	13.12	20.79	15.22

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## CONCLUSION

The paper focuses on the bi-level dynamic OD demand estimation problem for congested networks. The conventional assumption on the linear relationship between OD flows and link flows is discussed. This assumption does not hold in congested networks due to the complex interaction of OD flows on link flows. Therefore in this study, the conventional assumption is relaxed but with a further requirement on the sensitivity analysis of all OD pairs to link flows. Since deriving the sensitivity of all OD pairs makes the approach impractical for real-size networks, a heuristic solution is proposed that can avoid the extra computation burden. The results from a toy network and two real networks suggest that the exact computation of the sensitivity matrix is inefficient. The heuristic method proposed in the study has successfully estimated the target demand, as well as the existing method in the literature but with much smaller execution time, which can be very beneficial in analyzing large scale networks. The case studies presented in the paper were implemented at the microscopic level. For mesoscopic and macroscopic simulations with thousands of links and numerous traffic zones, further computational improvements are required. One of the main advantages of the proposed approach is the high level of parallelism. The extra assignments can be conducted concurrently, which is a desirable technique to speed up the solution algorithm. In addition, relying only on the observed traffic volume can be considered as a limitation for this study. The proposed model can adapt a multi-objective framework where various types of traffic data sources can be synthesized in a large-scale congested network. A possible extension is to use multi-source data to estimate more accurate OD demand matrices particularly for highly congested networks. Another possible extension is to combine the proposed heuristic solution with more efficient DNL models to improve the computational performance in large-scale network applications.

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