Estimating Network Fundamental Diagram using Three-Dimensional Vehicle Trajectories: Extending Edie’s Definitions of Traffic Flow Variables to Networks

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Estimating Network Fundamental Diagram using Three-Dimensional Vehicle Trajectories: Extending Edie’s Definitions of Traffic Flow Variables to Networks

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Abstract

This paper evaluates different measurement methods of traffic flow variables taken at the network level. Generalized Edie’s definitions of fundamental traffic flow variables along highways are extended to consider vehicles traveling in networks. These definitions are used to characterize traffic flow in networks, and form the basis for estimating relations amongst network density, flow and speed, in the form of a Network Fundamental Diagram (NFD). The method relies on three-dimensional vehicle trajectories to provide estimates of network flow, density, and speed. Such trajectories may be routinely obtained from particle-based microscopic and mesoscopic simulation models, and increasingly becoming available from tracking devices on board vehicles. Numerical results from the simulation of two actual networks, Chicago and Salt Lake City, are presented to illustrate and validate the estimation methodology. As part of the verification process, the study confirms that, as expected, the traffic flow fundamental identity \((Q=K.V)\) holds at the network level only when network-wide traffic flow variables are defined consistently with Edie’s definitions.
INTRODUCTION

The origins of network traffic flow theory can be traced to the 1960’s. Smeed (1, 2), Thomson (3), Wardrop (4), Godfrey (5), and Zahavi (6) were among the first studies to explore macroscopic relations of vehicular traffic in a network. In the late 1970’s, a two-fluid theory of town traffic was developed by Herman and Prigogine (7), who proposed a relation between the average fraction of vehicles moving in a street network and their average speed. In the 1980’s, the two-fluid theory was empirically validated for several cities of the world by Chang and Herman (8), Herman and Ardekani (9), and Ardekani and Herman (10), primarily using data obtained by chase-car techniques and aerial photography. Mahmassani et al. (11,12) and Williams et al. (13,14) extended the network-level variables, relating average speed, flow, and concentration by taking averages over all vehicles in the network over a specified time period. For the first time, Mahmassani and Peeta (15) observed a hysteresis pattern, “two-phase phenomenon”, in the average network density and average network speed relationship using simulation data for an urban network. See Figure 1(a).

Daganzo (16) re-theorized the network-level macroscopic relationships and recognized that if network-level macroscopic relations are insensitive to origin-destination (O-D) demand, they could be viewed as properties of the network. Geroliminis and Daganzo (17) subsequently showed that a characteristically-shaped flow-density relation indeed exists for a complete network using loop detector and GPS data from Yokohama (Japan). See Figure 1(b). Moreover, several recent studies (18-23) have shed light on the properties of network-wide flow-density relations. Such network-wide relationship has been referred to as “Macroscopic Fundamental Diagram” (MFD) or “Network Fundamental Diagram” (NFD).

![Figure 1](image-url)

**FIGURE 1 (a) Observed “Two-Phase Phenomenon” in the Network Speed-Density Relationship (15) and (b) NFD of Yokohama, Japan (18)**

Most of the related studies to date (19-23) have used the classical link-based measurement method, proposed in Mahmassani et al. (11), to estimate the NFD by taking the distance-weighted (or in some studies, unweighted) averages of flow and density over all the links in the network. Alternatively, it is possible to extend Edie’s (24,25) well-known generalized variable definitions of vehicle traffic flow along a highway to a network, as recently recognized by Courbon and Leclercq (26). With growing availability of trajectories from particle-based microscopic and mesoscopic simulation models, as well as from vehicle-based tracking devices, building variable definitions, NFD’s and other network-level characterizations on individual vehicle trajectories holds considerable theoretical and operational appeal (27,28). This paper operationalizes and validates the extension of Edie’s definitions to the network level, using it to demonstrate a trajectory-based measurement method to estimate the NFD. The proposed method uses three-dimensional (3D) vehicle trajectories in time and space to estimate network flow, density, and speed.
The remainder of the paper is organized as follows. Next section provides a background on NFD estimation methods. The third section provides a complete description of the link-based and trajectory-based measurement definitions of network traffic flow variables. The fourth section compares the two estimation methods using simulation results of the Chicago and Salt Lake City networks. Section five concludes the paper and suggests directions for future research.

**BACKGROUND**

The above-noted study by Courbon and Leclercq (26) compared three different methods for estimating NFD’s: (i) analytical method, (ii) “production” (or trajectory-based) method and (iii) loop detector (or link-based) method. The first is based on Daganzo and Geroliminis (29) and Boyaci and Geroliminis (30), who obtained analytical NFD expressions for idealized theoretical networks under spatially homogeneous congestion distribution. They used the variational theory and a moving observer across the network to derive an upper bound for the NFD. Boyaci and Geroliminis (31) extended the method to multi-modal networks. Most recently, Leclercq and Geroliminis (32) improved the analytical method to estimate NFD in a theoretical network with multiple signals and parallel hyperlinks.

In Courbon and Leclercq’s (26) comparison of the trajectory-based method with other estimation methods, they defined network flow and density as the following:

\[
\tilde{q}(t \to t + \Delta t, x \to x + \Delta x) = \frac{\sum_i l_i}{\Delta t \Delta x} \tag{1}
\]

\[
\tilde{k}(t \to t + \Delta t, x \to x + \Delta x) = \frac{\sum_i t_i}{\Delta t \Delta x} \tag{2}
\]

where \(l_i\) and \(t_i\) are respectively the distance traveled and the time spent by vehicle \(i\) in a time-space area of \(\Delta t \cdot \Delta x\). Courbon and Leclercq (26) recognized that the main difficulty of using the trajectory-based method is “the definition of the space-time window”, and accordingly did not operationalize a complete derivation of the network-wide Edie’s definitions for an actual network.

This paper builds upon the above previous studies and introduces a 3D time-space shape of \(\Delta t \cdot \Delta x \cdot \Delta y\) which is no longer a plane. Using the defined 3D time-space shape with 3D trajectories, Edie’s generalized definitions of traffic flow variables are extended to networks, and operationally demonstrated in establishing NFD’s for two actual networks using simulated trajectories. The paper demonstrates that a fundamental methodological issue in the 3D definitions is to recognize the topological features of the network, which does not occupy the entirety of the \(\Delta x - \Delta y\) plane, and hence limits the spatial range of the vehicle trajectories. The paper presents further illustration and validation of the approach through numerical results using simulate trajectories from actual urban networks with complex configuration and control settings.

**DEFINITION OF NETWORK TRAFFIC FLOW VARIABLES**

Network-wide traffic flow variables are commonly computed based on link measurements. Link-based measurements are estimated when traffic data from sensors installed on individual links of the network are available. Here we demonstrate that network-wide traffic flow variables can also be computed using individual vehicle trajectories. With the development and deployment of a fully connected transportation system, known as connected vehicle technology, individual vehicles act as “probes” in the system. Therefore, individual vehicle trajectories are expected to be widely available in future. The use of crowd-sourced vehicle trajectories is still limited but it is expected to grow in the next few years. Therefore,
network-wide trajectory-based measurements can be estimated when individual vehicle trajectories are available either through probe vehicles, crowd-sourced traffic data, or simulation.

**Link-based Measurements**

To estimate network-wide link-based measurements, the average network flow, average network density, and average network speed can be computed as follows, as derived by Mahmassani et al. (11):

\[
Q = \left( \frac{\sum_{i=1}^{M} l_i q_i}{\sum_{i=1}^{M} l_i} \right) \\
K = \left( \frac{\sum_{i=1}^{M} l_i k_i}{\sum_{i=1}^{M} l_i} \right) \\
V = \left( \frac{\sum_{i=1}^{M} l_i v_i}{\sum_{i=1}^{M} l_i} \right)
\]

where \( Q \) is the average network flow, \( K \) is the average network density, \( V \) is the average network speed, \( q_i \), \( k_i \), and \( v_i \), respectively, are the individual link average flow, density, and speed over the observation period and \( l_i \) is the length of lane-link \( i \), \( i=1, \ldots, M \), where \( M \) denotes the total number of lane-links. These measures are distance-weighted averages over the entire network. Here, we recognize that the traffic flow fundamental identity, \( Q=K.V \), would hold only approximately because the network speed and flow are not defined to reflect the space-mean values consistently with Edie’s definitions.

**Trajectory-based Measurements**

On individual road facilities, vehicles are commonly constrained to move along a one-dimensional path (e.g. a highway segment). Therefore, time-space diagrams of individual facilities usually include only two axes of time \( t \) and distance \( x \), as in Figure 2. Edie (24,25) proposed generalized definitions of flow, density, and speed for individual facilities (or road segments), which can be calculated in a two-dimensional (2D) time-space diagram as the following:

\[
q(A) = \frac{d(A)}{|A|} \\
k(A) = \frac{t(A)}{|A|} \\
v(A) = \frac{d(A)}{t(A)}
\]
where \( q(A) \), \( k(A) \), and \( v(A) \) are flow, density, and speed for observed vehicles in region A; \( d(A) \) is the total distance traveled by all vehicles in region A; \( t(A) \) is the total time spent by all vehicles in region A; and \( |A| \) is the area covered by region A as shown in Figure 2 (shaded area). In a seminal contribution to modern traffic analysis, Cassidy and Coifman (33) offered a deeper understanding of Edie’s definitions and their implications for correct interpretation of sensor data commonly available along freeways.

\[
\text{FIGURE 2 2D Trajectories in a Time-Space Diagram}
\]

Unlike the common application of 2D time-space diagrams when studying vehicle flow on individual facilities, network traffic has a 3D time-space diagram. In a network, vehicle movements from an origin to a destination are complex and can cover a wide range of \( x \) and \( y \) in the \( x-y \) plane. See examples in Figure 3(a). In order to estimate network-wide traffic flow variables using trajectories, we use a closed 3D shape \( \omega \), for example a cube, similar to region A in the 2D time-space diagram. Note that the 3D shape can be of any form and not necessarily be a cube. Also note that the desired network structure is laid down on the \( x-y \) plane as shown in Figure 3(b). Figure 4 illustrates simulated 3D trajectories of 1,000 vehicles in Irvine, California network as an example.

\[
\text{FIGURE 3 (a) 3D Trajectories in a Time-Space Diagram and (b) Network Structure on the x-y Plane of the 3D Time-Space Diagram}
\]
Therefore, the generalized network-wide traffic flow variables based on the extended Edie’s definitions can be expressed as follows:

\[ Q(\omega) = \frac{d(\omega)}{L_{xy}(\omega) \times \Delta t} \]  
\[ K(\omega) = \frac{t(\omega)}{L_{xy}(\omega) \times \Delta t} \]  
\[ V(\omega) = \frac{d(\omega)}{t(\omega)} \]

where \( Q(\omega) \), \( K(\omega) \), and \( V(\omega) \) are the network-wide average flow, density, and speed for the specified shape \( \omega \); \( d(\omega) \) is the total distance traveled by all the vehicles in the shape \( \omega \), \( t(\omega) \) is the total time spent by all vehicles in the shape \( \omega \), \( L_{xy}(\omega) \) is the total length (in lane-miles or lane-kms) of the network on the \( x-y \) plane associated with the shape \( \omega \), and \( \Delta t \) is the time height of the shape \( \omega \).

Similar to the conventional definition of density, network-wide density can be simply expressed as the number of vehicles in the network \( n_t \) at any time \( t \) divided by the lane-miles of the network as shown in Figure 5(a):

\[ K(t) = \frac{n_t}{L_{xy}} \]
Also, such illustration of network-wide vehicle trajectories allows defining consistent network-wide cross-sectional flows in two directions as shown in Figure 5(b). For any cross section of $dx$ or $dy$, the cross-sectional flow of each direction for any time interval $\Delta t$ can be expressed as:

$$Q_c(x) = \frac{n_x}{\Delta t}$$ (13)

$$Q_c(y) = \frac{n_y}{\Delta t}$$ (14)

Where $n_x$ and $n_y$ are number of vehicles on the $dx$ and $dy$ plane, respectively. The definitions used in constructing the NFD’s in the next section are those given by Equations 9-11, consistently with Edie’s extension to the network level.

**NUMERICAL RESULTS**

This section presents numerical results to demonstrate and validate the operationalization of Edie’s definitions for network flow, and compare it to the link-based measurement method. Networks of Chicago and Salt Lake City are simulated in a simulation-based dynamic traffic assignment platform (DYNASMART-P) with normal daily demand and 20% adaptive drivers to prevent formation of large gridlock (34). Outer suburbs are excluded from the networks for better computational efficiency. See Figure 6. Note that the temporal aggregation interval is 5 minutes for all the plotted figures in this section. The relationship between the number of vehicles in the network and trip completion rate (per 5 minutes) of the simulated networks are plotted and shown in Figure 7. In both networks, during the unloading period a hysteresis loop is formed. Also, at the end of the simulation a relatively small gridlock remained undissipated (22). Additional details on the network building and calibration aspects are provided in Refs. (36,37).
In the link-based method, individual link flows are obtained at the mid-point of each link and individual link densities are obtained by dividing the average number of vehicles on the link by the length-lane of the link. Trajectory-based flows and densities are estimated using Equations 9 and 10. However, when using simulated vehicle trajectories, some “edge-effects” (26) may create a bias in the estimated network flows and densities. The bias is mostly due to: (i) waiting time at the generation link, and (ii) incomplete trips at the end of the simulation. The boundaries of the considered 3D shape need to be carefully selected when working with simulated 3D trajectories. The estimated value of network-wide variables largely depends on where the edges of the 3D trajectories are. The waiting times at the generation link and destination nodes must be removed from the vehicle trajectories prior to the estimation of network-wide variables.
Figure 8 compares the estimated trajectory-based NFD versus the estimated link-based NFD for the selected networks when the waiting time at the generation link is removed from the trajectory of each simulated vehicle. In both NFDs, a large hysteresis loop is formed due to the inhomogeneous congestion distribution during the unloading (recovery) period. Figures 9 and 10 provide further visual comparison of network densities and flows with regard to the estimation methods. In both networks, for network densities greater than 20 vpmpl, the link-based method underestimates average network densities. However, both estimation methods yield near-identical network flows. When densities are high, the link-based method does not fully capture the variability of the congestion effects in the network. Also, averaging the number of vehicles on individual links in each time interval creates a bias in the link-based method when estimating network densities.

FIGURE 8 NFDs of (a) Chicago Network and (b) Salt Lake City Network with Different Measurement Methods

FIGURE 9 Comparison of Different Measurement Methods in Estimating Network Flow and Density for the Chicago Network
FIGURE 10 Comparison of Different Measurement Methods in Estimating Network Flow and Density for the Salt Lake City Network

Traffic Flow Fundamental Identity

Here we demonstrate that the network traffic flow fundamental identity, $Q=K.V$, holds when trajectory-based measurements are used. The traffic flow fundamental identity holds as long as all the network-wide variables are estimated using the same 3D shape. Figure 11 presents the plots of trajectory-based network flows versus $K.V$. As expected, results confirm that the traffic flow fundamental identity holds at the network level, when the network traffic flow variables are measured and defined consistently with the generalization of Edie’s definitions.

FIGURE 11 Trajectory-based Network Flow versus $K.V$ for (a) Chicago and (b) Salt Lake City Networks

CONCLUSION

This paper extends the generalized Edie’s definitions of fundamental traffic flow variables along highways to the network level. The extended definitions are used to estimate the NFD using three-dimensional vehicle trajectories. Such trajectories may be obtained from particle-based simulation models,
and increasingly becoming available from tracking devices on board vehicles. This paper builds on Courbon and Leclercq’s approach (26) and introduces a 3D time-space shape of $\Delta t.\Delta x.\Delta y$. Using the defined 3D time-space shape with 3D trajectories, Edie’s generalized definitions of traffic flow variables are operationally demonstrated in establishing NFD’s for two actual networks, Chicago and Salt Lake City, using simulated trajectories. The paper demonstrates that a fundamental methodological issue in the 3D definitions is to recognize the topological features of the network and hence limits the spatial range of the vehicle trajectories. The paper presents further illustration and validation of the approach through numerical results. As part of the verification process, the study confirms that, as expected, the traffic flow fundamental identity ($Q=K.V$) holds at the network level when network-wide traffic flow variables are defined consistently with Edie’s definitions.

With the proposed estimation method, the hysteretic behavior of network traffic flow and notion of “two-dimensional NFD” needs to be revisited. The “two-dimensional NFD” relates average network flow, average network density, and standard deviation of link densities (22). Since in the trajectory-based method no averaging is performed, heterogeneity of congestion distribution, as the main cause of hysteresis phenomena, is not represented by the standard deviation of link densities anymore. Thus, alternative measures using trajectories need to be developed to measure heterogeneity of congestion distribution in the network.

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